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THE NORSE TREATISE ALGORISMUS

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ABSTRACT. *Algorismus* is a treatise on Hindu-Arabic numerals, composed in the 13th century and found in four manuscripts dated 1300–1550. It is a translation of *Carmen de Algorismo*, a hexameter by Alexander de Villa Dei. Here, additions in *Algorismus* to *Carmen* are examined and the four manuscripts are compared by a numerical method.

Algorismus

The thirteenth century treatise *Algorismus* has been preserved in manuscripts, written in the Norse language, spoken in Iceland and Norway in the Middle Ages. The bulk of the treatise is a prose translation of the Latin hexameter poem *Carmen de Algorismo*, written in France in the early thirteenth century. The poem and the treatise introduce the Hindu-Arabic number notation to Europeans, the treatise to Icelanders and Norwegians in particular.

Algorismus exists in four manuscripts, AM 544, 4to, AM 685, 4to, AM 736 III, 4to, preserved in Copenhagen, and GKS 1812, 4to, preserved in Reykjavík.

In this paper we explore incidences where *Algorismus* deviates from *Carmen de Algorismo* and compare the four extant manuscripts of *Algorismus* numerically.

Algorismus was first published in a scientific edition 1892–1896 by Finnur Jónsson. The basic manuscript used was AM 544, 4to, corrected using the three other manuscripts when applicable. The Norwegian mathematician Otto B. Bekken translated *Algorismus* into modern Norwegian in 1985 and explained its text in cooperation with linguist Marit Christoffersen (Bekken & Christoffersen, 1985). Kristín Bjarnadóttir (2002; 2004; 2007, pp. 43–47) has explained the content of *Algorismus* in English and modern Icelandic. Helgi Guðmundsson (1967) has conjectured that Algorismus was composed in Viðey monastery, on Viðey Island off Reykjavík.

The contents of Carmen de Algorismo and Algorismus

Algorismus contains an explanation of the Hindu-Arabic number notation, including its place value and seven arithmetic operations: addition, subtraction, doubling, halving, multiplication, division and extraction of roots: the square root and cubic root. These methods have been transferred to *Algorismus* via a well-known Latin hexameter, *Carmen de Algorismo*, written by the Frenchman Alexander de Villa Dei between 1200 and 1203 (Beaujouan, 1954, p. 106).

The manuscript MS. Auct. F.5.29, containing *Carmen de Algorismo*, preserved in the Bodleian Library in Oxford, dated to the thirteenth century, has been compared to *Algorismus* in the manuscript AM 544, 4to. The comparison reveals that *Algorismus* is a direct translation of *Carmen de Algorismo*, found in MS. Auct. F.5.29. Both manuscripts have chapter headings which are found neither in the other manuscripts of *Algorismus* nor in printed versions of *Carmen* (Steele, 1988; Halliwell, 1841).

Carmen de Algorismo is an extraction of Muhamed ibn-Musa al-Kwarizmi’s *Kitab al-jam´val tafriq bi hisab al-Hind* (1992). *Carmen* was one of the first works introducing Hindu-Arabic number notation and arithmetic to Europeans. The poem exists in a great number of manuscripts, preserved in libraries in France, Great Britain, the Netherlands and many other countries, and is considered to have played a greater role in distributing Hindu-Arabic number notation in Northern Europe than the well-known *Liber abaci* by Leonardo da Pisa.

The arithmetic operations of addition, subraction and division, explained in *Carmen* and *Algorismus*, are largly similar to present methods used in paper-and-pencil arithmetic. Multiplying two composite numbers, however, proceeds from the left, as opposed to common modern algorithms. The numbers to be multiplied are arranged so that the digit farthest to the right of the multiplicand is placed below the first digit (from left) of the multiplier. The multiplicand is multiplied by this digit which then disappears under the product. Then the multiplicand is moved one place to the right so that the rightmost digit is placed below the second digit of the multiplier in the upper row and the lower number is now multiplied by this in the same manner. The product is written over the multiplying digit and added to the next digits to the left as before.

Carmen and *Algorismus* do not illustrate their algorithms on the four arithmetic operations by examples. The following example is made up for clarification in this paper:

Multiply 523 by 217:

First 523 is multiplied by 2 and 2 disappears under the product:

217	**1046**17
523	523

The first digit on the right of the lower number is then moved one place to the right below the second digit of the upper number which now is the multiplying digit. In this example the digit 3 is to be placed below 1 and other digits similarly. Now 523 is multiplied by 1. It is not quite clear if the digits of the product should be added as they are calculated or afterwards all at the same time:

52	
104637	**10983**7
523	523

In the next step the digit 3 of the multiplicand is moved below 7 and the other digits similarly. Then 523 is multiplied by 7:

	35		**14**	**2**	
109837	1**0983**7	**11333**7	**11333**7	**113471**	**113491**
523	523	523	523	523	523

The order of the multiplications in a product of two three digit numbers is as follows:

$$(100a + 10b + c) \quad \text{multiplied by}$$
$$(100d + 10e + f)$$
$$= 100a\cdot100d + 100a\cdot10e + 100a\cdot f + 10b\cdot100d + 10b\cdot10e + 10b\cdot f + c\cdot100d + c\cdot10e + c\cdot f$$

The advantage of multiplying from the left is that the product of the digits can be added to the previous product as they are found and it is not necessary to carry.

Deviations of Algorismus from Carmen de Algorismo

Carmen de Algorismo is a poem to be read aloud and thus a verbal work. The beginning of the MS. Auct. F.5.29 reads as follows:

Hec algorismus ars presens dicitur ; in qua
Talibus Indorum fruimus bis quinque figuris
0 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1 ·
Primaque significat unum : duo vero secunda :
Tercia significat tria : sic procede sinistre
Donec ad extremam venias, que cifra vocatur ;

The ten digits in the third line of the poem are the only incidence where the new Hindu-Arabic digits are presented in *Carmen*. Everywhere else numbers are expressed in words. The poem explains algorithms which are now common but it does not show any examples. It is not known how the poem was used as an aid to computation but one may assume that calculations were made on tablets or a flat surface, strewn with sand, or else on a wax tablet.

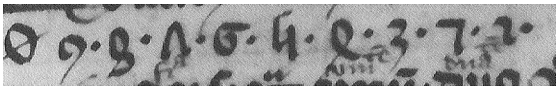

Figure 1: The ten digits of Hindu-Arabic numerals in Carmen de Algorismo in MS. Auct. F.5.29 (Bodleian Library).

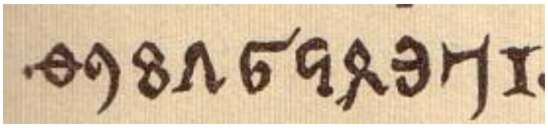

Figure 2: The ten digits of Hindu-Arabic numerals in Algorismus in AM 544, 4to (Jónsson, 1892–1896, p. 417).

Carmen de Algorismo is believed to be the first work where the zero, cifra, is presented as a digit (Beaujouan, 1954, p. 106).

The treatise *Algorismus* reacts to *Carmen's* lack of demonstration of the new system's number notation. It adds extensions in the first chapters, which point to a need to clarify the text by some numerical examples. Already after *Carmen* explains the place value notation, examples are inserted in the Icelandic translation.[1]

Ergo, proposito numero tibi scribere, primo
Respicias quis sit numerus ; que si digitus sit,
Primo scribe loco digitum [**this way, 8**]; si compositus sit
Primo scribe loco digitum post articulum sit [**as here, 65**]
Articulus si sit, cifram post articulum sit [**such, 70**]
(Jónsson, 1892–1896, p. 417).

Next even and odd numbers are presented, where the following addition is inserted in *Algorismus*:

Quolibet in numero, si [**it is a multiple of ten or**] par sit prima figura,
Par erit et totum, quicquid sibi continuetur;
Impar si fuerit, totum sibi fiet et impar.

Furthermore *Algorismus* adds for explanation that

> **even digits are four, 2, 4, 6 and 8, and uneven [odd] are another four, 3, 5, 7, 9. But one is neither as it is the origin of number** (Jónsson, 1892–1896, p. 418).

The digits are written in Hindu-Arabic mode in all extant manuscripts. *Algorismus* adds that one is neither even nor odd number as it is the origin of all number. Bekken et al. (1985, p. 27) have pointed out likeness to the statement that one is not a number, in al-Kwarizmi's *Arithmetic*, which again refers to another book on arithmetic, most likely either Euclid's Elements, book VII, or *Arithmetica* by the Neo-Pythagorean Nicomachus. The citation referred to is the following from the translation *Dixit Algorizmi* of al-Kwarizmi's work:

> J'ai déjà expliqué dans le livre d'algebr et almucabalah, c'est-à-dire de reprise et de rejet, que tout nombre est composé et que tout nombre est formé sur l'unité. L'unité se trouve donc dans tout nombre. Et c'est ce qui est dit dans un autre livre d'arithmétique, que l'unité est l'origine de tout nombre et existe en dehors d'un nombre (Allard, 1992, p. 1);

> I have already explained in the book on algebra and almucabalah, that is on restoring and comparing, that every number is composite and every number is composed of the unit. The unit is therefore to be found in every number. And this is what is said in another book on arithmetic that the unit is the origin of all numbers and is outside numbers.

Next time *Algorismus* adds an example, is when Carmen's text states that there are seven operations: addition, subtraction, doubling, halving multiplication,

[1] In the following examples, the Latin text is taken from the Carmen-manuscript MS. Auct.F.5.29, while the Old Norse additions are taken from Jónsson's edition of Algorismus, and translated into English. Translations from the Old Norse language were made by the author, K. B.

division and root extraction. Then *Algorismus* states that root extraction has two forms, extracting square root and cubic root:

> ... and is that branch in two ways. One is to take a root from a square number but the other to take a root from an eight-vertex number, which grows cubically (Jónsson, 1892–1896, p. 418).

Another elaboration of al-Kwarizmi's work by Sacrobosco, *Algorismus Vulgaris*, says: "... extraction of roots, which is twofold, since [it applies] to square numbers and cube numbers." (Grant, p. 95). This quotation points to that the translator of text may have known Sacrobosco's text in addition to Villa Dei's *Carmen*. Sacrobosco claims, however, that there are nine species of arithmetic operations, adding numeration and progression as operations no. one and eight.

Each arithmetic operation is explained in a separate chapter. To multiply, the reader is instructed to arrange the two numbers to be multiplied in columns such that the first digit (from the right) of the multiplier is placed below the last digit of the multiplicand. However, first one must check the difference of the larger digit of the multiplicand from ten and then delete the smaller one from its tens as often as that difference:

> In digitum cures digitum si ducere, major
> In quantum distet a denis respice, debes
> Namque suo decuplo tociens delere minorem;

Algorismus adds this explanation:

> **So that you understand this multiply vii and nine. Nine differs by one from x. Therefore take one vij from vij tens. Then remain iij and vi tens, that is vij times nine** (Jónsson, 1892–1896, p. 420).

In modern notation this may be written

$$7 \cdot 9 = 10 \cdot 7 - 1 \cdot 7 = 63 \qquad \text{or}$$

$a \cdot b = 10a - (10 - b)a \ \ (0 < a, b < 10)$.

Two conclusions may be drawn from this explanation. First, the Latin text is not considered to be clearly presented so that an example is needed. Second, the example demonstrates that as the translator/transcriber is not familiar with Hindu-Arabic digits, he uses Roman numerals and words. Numerals do not have a consistent representation across manuscripts, in the youngest manuscript some numbers are written in the Hindu-Arabic mode.

The multiplication example is the last one added for clarification in *Algorismus*, while several repetitions in *Carmen* were omitted in the translation.

Finally, a separate chapter is added to the translation on the cubic numbers 8 and 27 and their intermediate numbers 12 and 18, and their relation to the elements: Earth, $2^3 = 8$; Water, $2^2 \cdot 3 = 12$; Air, $2 \cdot 3^2 = 18$; Fire, $3^3 = 27$. This chapter does not exist in *Carmen*, and its content is unrelated to the main bulk of *Algorismus* in modern understanding. It says that it has been found necessary to add something in between the Earth and the Fire to unite them in their disagreement. Therefore, Water has two parts from Earth and one from Fire, and Air has one part from Earth and two from Fire. This puts the Elements in the correct order by lightness: Fire (27), Air (18), Water (12) and Earth (8) (Jónsson, 1892–1896, p. 423–424).

This produces the sesquialteral progression 8:12::12:18::18:27 or n:n+½n. This sequence of proportions and elements appears in St. John's College MS 17 (Oxford Digital Library), and in a similar schema in an eleventh century manuscript of Boethius, Madrid Biblioteca nacional Vit. 20 fol. 54v, and in a thirteenth c. Macrobius manuscript British Library Arundel 399 fol. 12v. It is also found in the anonymous treatise on cosmology in Bodleian Library Digby 83, fol. 3r (The Calendar and the Cloister – St. John's College MS 17, commentary; Bekken, 1986, p. 16).

The four manuscripts of Algorismus

The texts of *Algorismus* in the manuscripts AM 544, 4to, and GKS 1812, 4to, are identical in most respects, as is AM 685, 4to, which however has a 306 word long addition, not treated in this paper.

AM 544, 4to, is the oldest manuscript of the treatise, estimated to be written in the period 1302–1310, most likely in 1306–1308 (Karlsson, 1964, pp. 114–121). The text is divided into chapters which bear headings. Numbers are written using Hindu-Arabic numerals in the introduction and in the additions to *Carmen* by examples of place value notation and even and odd numbers shown earlier. Numbers are, however, mainly written by Roman numerals, until in the last section on the Elements, which does not belong to *Carmen de Algorismo*, and where Hindu-Arabic numerals are used.

The part of **GKS 1812, 4to**, containing *Algorismus* is estimated to be written in 1300–1400 (*A Dictionary of Old Norse Prose*, 1989– , p. 26). There are no chapter headings. Numbers are mainly written using words as in *Carmen de Algorismo*, exceptionally using Roman numerals and never using Hindu-Arabic numerals apart from in the first additions to *Carmen*, as is done in AM 544, 4to, and in the chapter on the Elements.

AM 685, 4to, is dated to 1450–1500 (A Dictionary of Old Norse Prose, 1989– , p. 26). It has no chapter headings. Number are written alternatively in words, Roman numerals and Hindu-Arabic notation, which is the most common. F. Jónsson states that the text of *Algorismus* in AM 685, 4to, is the most error free of the four texts, basing this conclusion on various spelling examples (Jónsson, 1892–1896, p. cxxxi). Furthermore this text is the most concise of the four texts as it is often contracted, preserving a correct meaning. In the following comparison the difference in spelling is not revealed as the texts of all the manuscripts have been rewritten to modern Icelandic. The text in AM 685, 4to, is also correct where other texts have an error on the origin of one half (Jónsson, 1892–1896, p. 419), called *semiss*, coming up after halving a number, which indicates that one of the transcribers of AM 685, 4to, understood the treatise well.

AM 736, 4to, is estimated to origin around 1550 (*A Dictionary of Old Norse Prose*, 1989–, p. 26). It contains only a fragment of the text of *Algorismus*, a section on root extraction. It does not contain the text on the Elements and their associated numbers, while on a different leaf in the same manuscript a diagram of the four Elements is found together with the Elements and the number xii associated to Water, xviii to the Air and *tres, trium, tres* to the Fire. Diagrams with the Elements and the four numbers exist in other manuscrips as cited earlier, but they are unrelated to *Algorismus* (Bekken, 1986, p. 16).

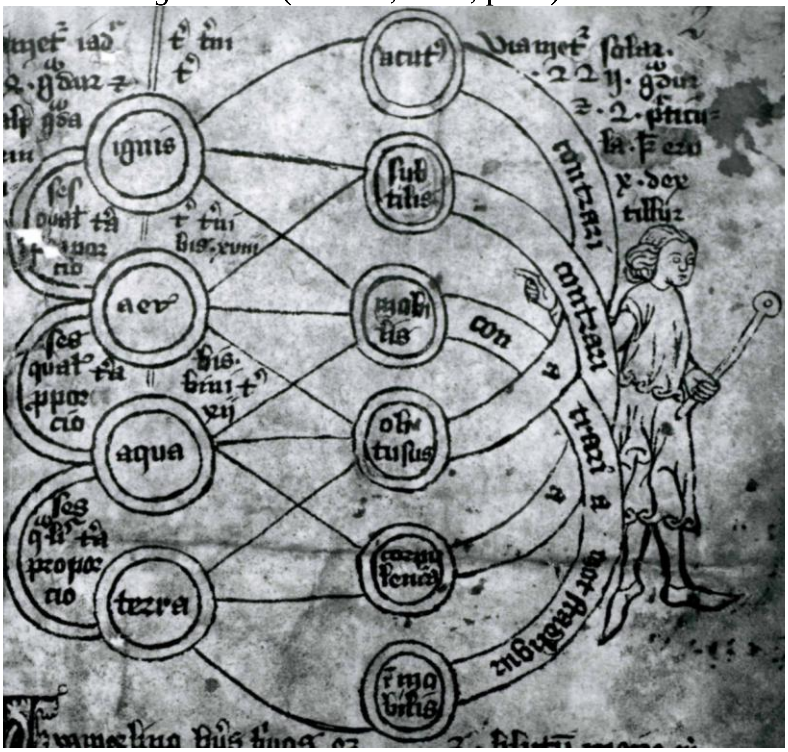

Figure 3. A diagram of the four Elements in the manuscript AM 736 III, 4to.

The adaptations made to *Carmen de Algorismo* to create *Algorismus* suggest that *Algorismus* served a role in introducing the use of Hindu-Arabic numerals in Iceland. In the oldest manuscript of *Algorismus*, AM 544, 4to, Roman numerals are used to explain the text or plain words as in *Carmen*. The use of Roman numerals suggests on one hand that the transcriber needed to shorten the text and on the other that he was not used to Hindu-Arabic numerals.

Plain words are dominant in GKS 1812, 4to. The youngest whole manuscript, AM 685, 4to, rarely has Roman numerals, while words and Hindu-Arabic numerals are used alternatively.

Manuscript comparison - methodology

When reading the four manuscripts of Algorismus it is apparent that they are quite similar; sentence structure and phrasing suggests that they all have the same root. The same text insertions and deletions are made in all four manuscripts to Carmen de Algorismo, exemplifying that these are not different translations. But how similar are these manuscripts?

Numerical methods were used to compare the manuscripts, comparable to methods that have been used extensively in comparative linguistics and in gene and protein comparison.

The four texts were aligned using the computer program ClustalW and a weighted number of mismatches between the manuscripts was computed. As ClustalW is designed to align protein sequences it takes as input sequences from the twenty letter alphabet of protein sequences. As the Latin alphabet is larger than twenty letters, each letter was mapped to two letters in the alphabet of protein sequences. ClustalW was then used to align the texts and the text was mapped back to the Latin alphabet. The alignment was then corrected manually, considering in particular word reorder and different forms of the imperative.

Mismatches between the manuscripts were counted and classified into three distinct classes; *single character mismatches*, *word reorders* and *word mismatches*. *Single character mismatches* were defined as:

- Spelling is identical apart from a single character difference.
- Mismatches in writing style of the numerals; Hindu-Arabic, Roman or spelled out.
- Mismatches in the writing of the imperative, e.g. tak þú - taktu.

Word reorders were defined as parts of the manuscripts where the order of two or more words had been reordered.

Word mismatches were all other types of differences such as word insertion, missing words or a different word being used.

The weighted distance between the manuscripts was used to infer the phylogeny of the manuscripts, using the assumption that it is unlikely that the same change is made more than once. One may also assume that each transcriber is equally likely to cause a distinction.

Finally, a simple program was written to count the number of differences.

Results

The manuscripts are different in length. In the following a section in AM 685, 4to, of length 306 words, not extant in the other manuscripts, has been removed. The lengths are:

Manuscript	Words #	Characters #
AM 685, 4to	2902	14772
AM 736 III, 4to	630	3323
AM 544, 4to	2960	15110
GKS 1812, 4to	2986	15174

Table 1. No. of words and characters in the four manuscripts of Algorismus.

That AM 685, 4to, has fewest words of the complete manuscripts confirms the reader’s intuition that the transcriber(s) of AM 685, 4to, sometimes shorten the text.

The following weights of mismatches were used:

Word mismatches: 1,00 point

Word reorders: 0,25 point

Single character mismatches: 0,25 point

Results from counting mismatches between the three complete texts in AM 685, 4to, AM 544, 4to, and GKS 1812, 4to, were:

Manuscripts	AM 685, 4to	AM 544, 4to	GKS 1812, 4to
AM 685, 4to	0,00	261,00	264,50
AM 544, 4to	261,00	0,00	123,25
GKS 1812, 4to	264,50	123,25	0,00

Table 2. No. of mismatches between the three complete manuscripts of Algorismus

The greatest distance between two manuscripts is between AM 685, 4to, and GKS 1812, 4to, 264,5 mismatches by 2986 words, or 8,9%.

The shortest distance between two manuscripts is between AM 544 4to, and GKS 1812, 4to, 123,25 mismatches by 2986 words, or 4,1%.

The parts of the manuscripts that they have all in common, that is the part also found in AM 736 III, 4to, were compared separately. The results were:

Manuscripts	AM 685, 4to	AM 736 III, 4to	AM 544, 4to	GKS 1812, 4to
AM 685, 4to	0,00	73,25	52,00	55,00
AM 736 III, 4to	73,25	0,00	57,25	57,75
AM 544, 4to	52,00	57,25	0,00	26,00
GKS 1812, 4to	55,00	57,75	26,00	0,00

Table 3. No. of mismatches in the part common to all manuscripts of Algorismus

The distance of AM 736 III, 4to, is greatest from AM 685, 4to, while it is closest to AM 544, 4to, and nearly equally close to GKS 1812, 4to.

Clearly, AM 544, 4to, and GKS 1812, 4to, are more close to each other than the other two, which are also different from each other.

In this counting of mismatches the ratio 1 : 0,25 or 4 : 1 between *word mismatches* and other mismatches was used. Counting was also done using the ratio 3 : 1 and lead to comparable conclusions.

Figure 3 exhibits the relation beween the different copies of *Algorismus*. A matrix was made according to the distances between the four manuscripts, from which was constructed a phylogenetic tree with distances similar to the distances in the distance matrix. The diagram was made by the program ATV (Zmasek and Eddy, 2001, p. 383 – 384).

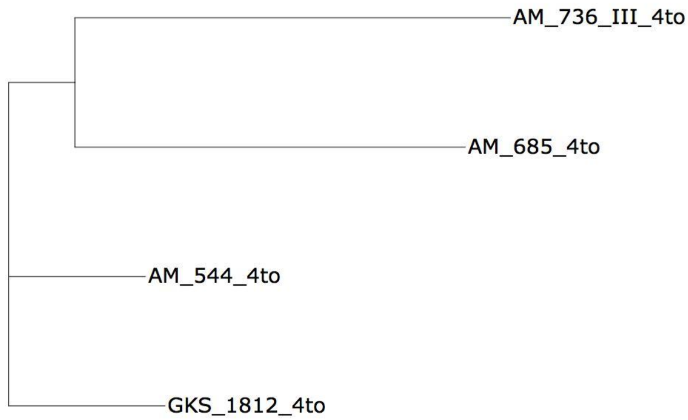

Figure 4. A phylogeny of the copies of Algorismus in the manuscripts AM 736 III, 4to, AM 685, 4to, AM 544, 4to, and GKS 1812, 4to, made by the program ATV.

The phylogeny may be interpreted such that the manuscript AM 544, 4to, contains the most original copy of the treatise, and that the copy in GKS 1812, 4to, is closest to it. The copies in the manuscripts AM 736 III, 4to, and AM 685, 4to, are partly drawn from the same stem, but are further from the origin, in particular AM 736 III, 4to.

Discussion

What motivated Icelanders to translate Carmen? Certainly, they had to count their belongings and assets. e.g. for taxes, but they could have done that with the Roman numerals they knew. Writing manuscripts was an integral part of the Christian monastic culture. The reason may have been an aspiration to belong to the European cultural world, while Latin was an alien language to the recently literate Germanic tribes that had settled in the Nordic countries. The population in Iceland was never large, around 50–70,000 in the 13th century. Producing writings in the vernacular was an important factor in creating a common culture of this small group of people.

The additions in Algorismus to its original, Carmen de Algorismo, bear witness to a desire for learning, to understand the text. Comparison of the four copies of Algorismus of different age reveals that people continued to work on understanding the text and gradually began to use the convenient Hindu-Arabic number notation.

But it took time. According to the phylogeny and other considerations, manuscript AM 544, 4to, was not the original of Algorismus, which suggests that Algorismus may have been written in the second half of the thirteenth century, or about 200 years before AM 685, 4to, and possibly up to 300 years before AM 736 III, 4to. Algorismus therefore played an important role in Icelandic culture until the era of printing, when printed books began to spread much more rapidly between countries than manuscripts.

Iceland was originally an independent society, but from 1397 it belonged to the Danish realm until 1944. Due to worsening living conditions and its colonial status, Iceland lagged behind other European countries in educational respects. Algorismus appears in history whenever mathematics education was revived, serving as a monument of the proud past, when Icelanders kept up with the latest global knowledge. Even the most distinguished Icelandic scholars continued to refer to Algorismus up until the nineteenth century, (see e.g. Gunnlaugsson, 1865, p. 4), paying respect to the time when Icelanders were familiar with the latest mathematical knowledge in the world and translated it to their own language.

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